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## Theoretical design of a self-rectifying 4-bar linkage mechanism

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**Abstract**

Mechanical systems will almost inevitably fail at some point during operation. This can either be due to a preexisting design flaw or some unexpected damage during usage. No matter how much planning and fault analysis is performed it is impossible to create a perfectly reliable machine. Existing approaches to improving reliability normally involve advances in modeling and detection to include specific mechanisms to overcome a particular failure or mitigate its effect. Whilst this has gone a long way to increasing the operational life of a machine, the overall complexity of systems has improved sharply and it is becoming more and more difficult to predict and account for all possible failure modes. Rather than focusing on mitigating or reducing the probability of failure, a new design philosophy is proposed that allows systems to reconfigure themselves to overcome failure – thus yielding a self-healing design. This approach is demonstrated in the design of a self-rectifying 4-bar linkage mechanism.

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**1. Introduction**

Bongard et al. stated that “most machines fail in the face of unexpected damage” [1]. Whilst this has yet to be unequivocally proven, it is a notion that agrees with intuition. If this is indeed the case, then the current approach to robust tolerant design is clearly not working. A possible reason for this, as stated by Thrun et al. is that one of the long-standing challenges is achieving robust performance under uncertainty [2]. Essentially, with increasingly complex systems, it is becoming more difficult to predict account for all possible modes of failure.

To address this issue, an alternative approach is suggested in which, rather than compensating for specific failure modes, a system is instead designed to be adaptable to any (non-catastrophic) failure – i.e. the system is capable of self-repair. To demonstrate this approach, this paper uses a simple example of a 4-bar linkage mechanism.

**1.1. Terminology**

It is difficult to discuss the concepts of self-healing and self-repair without first defining what is meant by self-healing. There isn't a universally-accepted definition of self-healing, but instead intuitive notions about the concepts involved. In general terms, what we are looking for are systems that are able to “Maintain some degree of functionality after a failure has occurred”. The primary cause of failure is not necessarily of interest, but it is assumed hereon that failure can and does occur. What is of more interest is how the system can adapt ‘post-failure’ to attempt to maintain functionality as close as possible to the intended design.

**1.2. 4-bar Linkage Mechanism**

Mechanical linkages, which are, at a basic level, an assembly of rigid elements connected via joints to translate motion or force, have been studied for several hundred years. Linkage mechanisms, and in particular 4-bar linkages, have

been used extensively in industry to transmit torque, motion and power, or to transform one type of motion or force to another [3]. From a theoretical point of view, 4-bar linkages have been most extensively covered in literature due to their relative simplicity, making analysis easier.

An example of a 4-bar linkage is shown in Figure 1. It consists of 4 links:

- Link 0 (input): This link is typically the driven link
- Link 1 (float): This link typically consists of a single, straight element, or a triangular rigid element
- Link 2 (output): This link can be used to rotate another element thus providing a transmission ratio
- Link 3 (fixed): This is the only link that is prevented from moving or rotating.

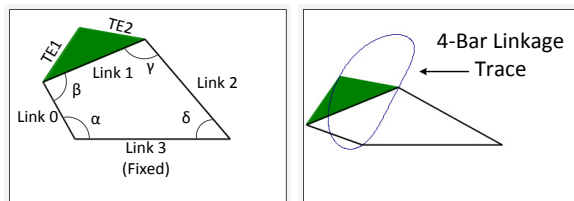


Fig. 1. Example of a 4-bar linkage mechanism and trace pattern

For the purposes of this study, it is assumed that the 4-bar linkage mechanism is designed to trace out a particular pattern. The apex of the triangular float link will follow a particular pattern when the input link is rotated one complete revolution. The specific pattern will depend on:

- The geometry (length) of the other links
- The geometry of the triangular float link, which can be completely described by the length of Link 1 and the two other tracer edge lengths (TE1 and TE2)

## 2. Approach to Designing Self-repairing Systems

To attempt to design a self-repairing system (or to apply self-repairing elements to existing systems), a generalized approach is proposed. This is partly based on [4], who states that what is desired is a system with “the ability to autonomously predict or detect and diagnose failure conditions, confirm any given diagnosis, and perform appropriate corrective intervention(s)”. This process, which can theoretically be applied to any system is broken down into four steps, as shown in Figure 2.

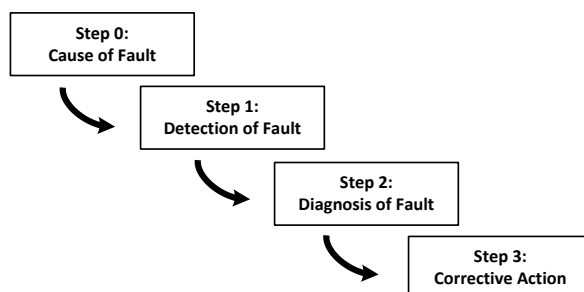


Fig.2. Proposed self-repair approach

### 2.1. Step 0: Cause of Fault

The underlying cause of the fault should not necessarily be the focus; else the approach would quickly degenerate into fault analysis. Instead the focus should be on how the fault manifests itself. To illustrate this point, the categorization scheme proposed by Collins [5] is used. According to this scheme, failures in mechanical systems have 3 main points of interest:

1. Failure inducing agents: force, time, temperature, reactive environment, human
2. Location of failure: Body, system or surface type
3. Manifestations of failure: Elastic deformation, plastic deformation, rupture or fracture, material change, etc.

#### 2.1.1. Failure inducing agent

Typically in a 4-bar linkage system under normal conditions, the failure inducing agent will likely be caused by mechanical load – either through vibration, or shock through impact. In extreme environments this list could be extended to include extreme thermal changes and perhaps chemical-based failure.

#### 2.1.2. Location of failure

In a linkage system there are three primary locations that failure can occur at a rigid element, a joint or an anchor point. There could also be a failure of the motion driving element (motor), but for the purposes of this example that is considered a separate system. In this instance the mechanics of self-healing for such a motor varies differently from the outlined 4-bar linkage system and therefore goes beyond the scope of this paper. This does however highlight the importance of careful selection of system boundaries. Since one of the goals of using this approach is to break a system down into fundamental elements, it is important to ensure there is some homogeneity between the elements in a sub-system. The same process can then be applied to a different sub-system (such as motor, tooling, etc.).

#### 2.1.3. Manifestation of failure

In each failure location, the failure-inducing agents can manifest in one of following ways:

1. Failure of rigid element:
  - 1.1. Deformation of body (extension, bending or twisting) – either by failure in stiffness of material or through thermal expansion/contraction
  - 1.2. Fracture/Split/Break of body – typically caused by mechanical load but might also be caused by corrosion or deterioration of material
  - 1.3. Obstruction of expected motion – case fails and prevents normal motion, or an element fails and interferes with other elements
2. Failure of joint
  - 2.1. Complete failure – disconnection of joint
  - 2.2. Range of motion limited – fundamentally changes behavior of mechanism, new dead-spots etc.
  - 2.3. Higher than expected resistance – increased frictional load, wear, etc.

2.4. Joint tolerances – ‘play’ in joint, adds additional DOF to mechanism

3. Failure of anchor point: Anchor point no longer restricts DOF of node as intended

Aside from those points above that would leave to catastrophic failure (disconnection of a joint/anchor, or a break in a rigid element), the other manifestations mentioned would lead to a change in the tracer pattern, which can essentially be characterized by a change in the linkage dimensions.

## 2.2. Step 1: Prediction or detection of fault

The manifestation of failure could perhaps be most easily interpreted as a deviation from the prescribed tracer pattern. There are a number of ways this could be inferred, such as:

- Deviation from expected behavior: System does something unexpected
- Externalized sensors: Independent test system to observe behavior, pressure, temperature, voltage, etc.
- User intervention: User reports fault (loses autonomy)

A reasonable approach for this problem is that the tracer pattern can be calculated from the expected linear element dimensions coupled with external sensors that, for example, measure the angle of each link ( $\alpha$ - $\delta$  in Figure 1).

## 2.3. Step 2: Diagnosis of Fault

Since it is assumed that the system knows that the tracer pattern is no longer following the designed trajectory, the next question to answer is how it can infer from that information, which element has caused this failure. There are several possible approaches to this, which can be summarized as follows:

- Model-based (Abductive reasoning): compare observation with predicted observation: I expect ‘X’ but get ‘Y’, therefore I must correct ‘Y’ to get it to match.
- Bayesian belief networks: probabilistic graphical model (a type of statistical model) that represents a set of random variables and their conditional dependencies: If ‘X’ and ‘Y’ happen, its likely a failure with ‘Z’
- Case-based reasoning methods: anecdotal evidence, if ‘X’ happens, do ‘Y’. – Accounts for expected failure only

Initially, a model based approach is proposed similar to what is shown in Figure 3. Using this approach, the system would operate as follows:

1. During normal operating all dimensions are known and the relationship between angles  $\alpha$ - $\delta$  is as expected
2. An element is damaged, and the angular relationship changes
3. The damaged element is remodeled as a straight element to compensate for the change in effective length
4. The system adapts itself to compensate

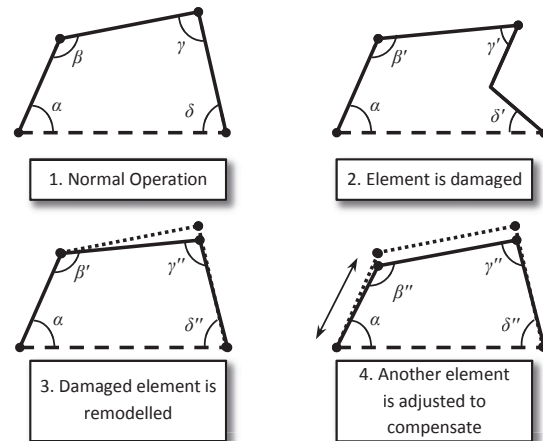


Fig. 3. Model based abductive reasoning

The drawback of this approach is that only one type of failure (rigid element deformation) is accounted for, which is a contradiction of the aims of designing a self-repairing system. However, the precise mathematical relationship between the angles and the link lengths are well established mathematically, thus providing sufficient evidence from which to infer a tracer pattern even if an element is changed. Hence for the purposes of this discussion, it is assumed the tracer pattern, both pre- and post-failure is known.

An alternative approach is thus proposed based on simplified Bayesian probability. Essentially the system asks itself:

*What do I need to remodel about myself to produce this new behavior?*

This approach has proven to be very successful in trials with the 4-bar linkage. Specifically the system is given the task of determining the least amount of changes required to the design parameters to produce the new tracer pattern. It can then use these new, remodeled parameters to determine what further action has to be taken to return to its modus operandi.

## 2.4. Step 3: Corrective Action

If it is assumed there is a failure in an element or joint play, then it can be reasonably assumed that we don't wish to affect these further. And in any case to replace the rigid element links with elements that can alter their dimensions (linear actuators for example) would reduce innate reliability. It is better therefore to change only the attached ‘tool’ – in this case a simple pen designed to draw a particular pattern. Hence, once the system has been remodeled with revised dimensions, it can then attempt to return back to the original desired path through changing dimensions of the tracer edges (TE1 and TE2 in Figure 1). This could be achieved through the automation of a very simply mechanism shown in Figure 4.

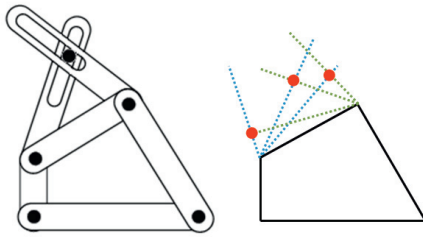


Fig. 4. Simple mechanism for varying floating element shape

The system thus attempts to solve a simple optimization problem in which the objective is to minimize the deviation between the designed tracer pattern and the new tracer pattern, which can be calculated as the average of the Euclidean distance between each point on the tracer function path at intervals of  $1^\circ$  changes in the input angle  $\alpha$ , as shown in Figure 5.

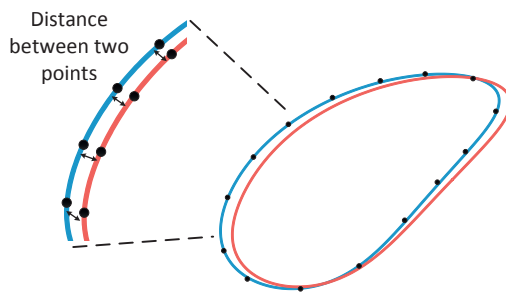


Fig. 5. Euclidean distance between tracer path points

### 3. Results

Using the methodology described, a 4-bar linkage is designed with dimensions shown in Table 1. Some failure event causes a change to the desired tracer pattern. Based on the new tracer pattern, the system is able to determine that Link 2 has changed its effective length to the new value shown in Table 1. The length of TE1 and TE2 are assumed to be unchanged and both equal to 3 units.

Table 1. 4-Bar linkage design and post failure dimensions

Link	As-designed Dimensions	Inferred Post-failure Dimensions
Link 0	3	3
Link 1	5	5
Link 2	6	5.75
Link 3	7	7

Given that the optimization problem is trivial (two inputs, single objective), a simple Generalized Reduced Gradient method is used to determine the new tracer edge lengths required to reduce the difference between the post failure and design tracer curves. Using this method, the system changed

the length of the tracer edges (TE1 and TE2) to the values shown in Table 2.

Table 2: Post-failure and rectified dimensions

Element	Post-failure Dimensions	Rectified Dimensions
Tracer Edge 1	3	3
Tracer Edge 2	3	3.1633

The effect of these changes on the tracer pattern is shown in Figure 6 as a function of the input angle  $\alpha$ . These differences are from the desired tracer pattern, hence a perfect solution would have a constant value of zero. It can be seen from this that a change of only 5% in TE2 has yielded a reduction in post-failure tracer error of almost 90%. The actual tracer patterns are shown in Figure 7. It can be seen from this graph that the rectified fix shows an almost perfect correlation with design (base) curve

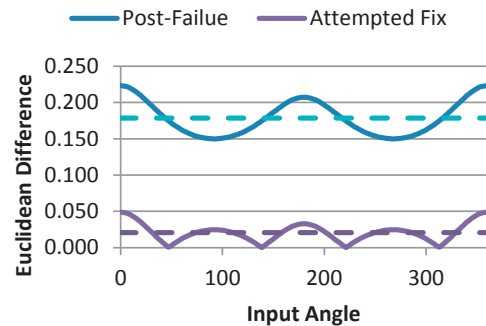


Fig. 6. Euclidean difference between designed tracer pattern and post-failure/attempted fix

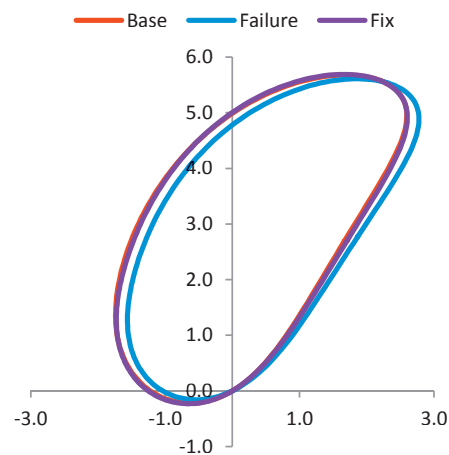


Fig. 7. Design, post failure and rectified tracer patterns

#### 4. Conclusions

This paper has used a simple example of a 4-bar linkage mechanism to demonstrate a possible approach to designing a self-repairing system. This was achieved by breaking the self-repair process down into four individual steps that can be applied to any system. Additionally a number of pitfalls were encountered:

- During Step 0: Cause of Fault: It is important to focus on a system that has homogeneity between elements. The rectification of a failed motor would be vastly different from the rectification of a rigid element. If a system has non-homogenous elements then it should be broken down into sub-systems and each one assessed individually
- During Step 1: Detection of Fault: Although the simplest way of detecting a fault would be through user intervention, this somewhat defeats the objective of having an autonomous system. A better alternative is to directly monitor the desired behavior to observe deviation. This information can later be used when attempting to rectify the fault
- During Step 2: Diagnosis of Fault: Although model-based reasoning is a tempting option, it generally leads the designer to only focus on particular, expected modes of failure. It is perhaps better to infer a possible, effective failure from a change in behavior
- During Step 3: Corrective Action: Where possible the designer should avoid introducing changes that would fundamentally alter the basic system mechanism. In this example, it might have been tempting to replace the rigid

linkages with deformable linkages that could alter their length (such as a linear actuator), however this would fundamentally change Steps 0-3, and the process would have to be repeated, leading to an endless design circle.

Systems with additional procedures built-in are invariably more complex and hence the primary system becomes intrinsically less 'reliable', even though it is able to bring itself back to a normal operating condition. However, looking at the overall concept of product reliability, if viewed from the perspective of the user, a system with an integrated self-repair feature would appear to be more 'reliable' – it is able to maintain operation for a longer period of time than would otherwise have been possible – and this is the primary aim of a self-repairing system.

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